Package ‘CompQuadForm’

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Title Distribution function of quadratic forms in normal variables
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Author P. Lafaye de Micheaux
Maintainer P. Lafaye de Micheaux <lafaye@dms.umontreal.ca>
Description Computes the distribution function of quadratic forms in normal variables using Imhof's method, Davies's algorithm, Farebrother's algorithm or Liu et al.'s algorithm.
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Description

Distribution function (survival function in fact) of quadratic forms in normal variables using Davies’s method.
Usage

davies(q, lambda, h = rep(1, length(lambda)), delta = rep(0, length(lambda)),
        sigma = 0, lim = 10000, acc = 0.0001)

Arguments

q                value point at which distribution function is to be evaluated
lambda           the weights $\lambda_1, \lambda_2, \ldots, \lambda_n$, i.e. distinct non-zero characteristic roots of $A\Sigma$
h                respective orders of multiplicity $n_j$ of the $\lambda$s
delta            non-centrality parameters $\delta_j^2$
sigma            coefficient $\sigma$ of the standard Gaussian
lim               maximum number of integration terms. Realistic values for lim range from 1000
                  if the procedure is to be called repeatedly up to 50 000 if it is to be called only
                  occasionally
acc               error bound. Suitable values for acc range from 0.001 to 0.00005 which should
                  be adequate for most statistical purposes.

Details

Computes $P\{Q > q\}$ where $Q = \sum_{j=1}^{r} \lambda_j X_j + \sigma X_0$ where $X_j$ are independent random variables
having a non-central $\chi^2$ distribution with $n_j$ degrees of freedom and non-centrality parameter $\delta_j^2$ for $j = 1, \ldots, r$ and $X_0$ having a standard Gaussian distribution.

Value

trace vector, indicating performance of procedure, with the following components:
1. absolute value sum, 2. total number of integration terms, 3. number of
   integrations, 4. integration interval in main integration, 5. truncation point in
   initial integration, 6. standard deviation of convergence factor term, 7. number
   of cycles to locate integration parameters

ifault fault indicator: 0: no error, 1: requested accuracy could not be obtained, 2:
          round-off error possibly significant, 3: invalid parameters, 4: unable to locate
          integration parameters

Qq $P\{Q > q\}$

Author(s)

Pierre Lafaye de Micheaux (<lafaye@dms.umontreal.ca>) and Pierre Duchesne (<duchesne@dms.umontreal.ca>)

References

P. Duchesne, P. Lafaye de Micheaux, Computing the distribution of quadratic forms: Further comparisons between the Liu-Tang-Zhang approximation and exact methods, Computational Statistics and Data Analysis, Volume 54, (2010), 858-862

Examples

# Some results from Table 3, Davies (1980)

round(1-davies(1,c(6,3,1),c(1,1,1))$Q,q,4)
round(1-davies(7,c(6,3,1),c(1,1,1))$Q,q,4)
round(1-davies(20,c(6,3,1),c(1,1,1))$Q,q,4)
round(1-davies(2,c(6,3,1),c(2,2,2))$Q,q,4)
round(1-davies(20,c(6,3,1),c(2,2,2))$Q,q,4)
round(1-davies(60,c(6,3,1),c(2,2,2))$Q,q,4)
round(1-davies(10,c(6,3,1),c(6,4,2))$Q,q,4)
round(1-davies(50,c(6,3,1),c(6,4,2))$Q,q,4)
round(1-davies(120,c(6,3,1),c(6,4,2))$Q,q,4)
round(1-davies(20,c(7,3),c(6,2),c(6,2))$Q,q,4)
round(1-davies(100,c(7,3),c(6,2),c(6,2))$Q,q,4)
round(1-davies(200,c(7,3),c(6,2),c(6,2))$Q,q,4)
round(1-davies(10,c(7,3),c(1,1),c(6,2))$Q,q,4)
round(1-davies(60,c(7,3),c(1,1),c(6,2))$Q,q,4)
round(1-davies(150,c(7,3),c(1,1),c(6,2))$Q,q,4)
round(1-davies(70,c(7,3,7,3),c(6,2,1,1),c(6,2,6,2))$Q,q,4)
round(1-davies(160,c(7,3,7,3),c(6,2,1,1),c(6,2,6,2))$Q,q,4)
round(1-davies(260,c(7,3,7,3),c(6,2,1,1),c(6,2,6,2))$Q,q,4)
round(1-davies(-40,c(7,3,-7,-3),c(6,2,1,1),c(6,2,6,2))$Q,q,4)
round(1-davies(40,c(7,3,-7,-3),c(6,2,1,1),c(6,2,6,2))$Q,q,4)
round(1-davies(140,c(7,3,-7,-3),c(6,2,1,1),c(6,2,6,2))$Q,q,4)

farebrother  Ruben/Farebrother method

Description
Distribution function (survival function in fact) of quadratic forms in normal variables using Farebrother’s algorithm.

Usage
farebrother(q,lambda,h = rep(1, length(lambda)),delta = rep(0,length(lambda)),
maxit=100000,eps=10^(-10),mode=1)

Arguments

q value point at which distribution function is to be evaluated
lambda the weights $\lambda_1, \lambda_2, ..., \lambda_n$, i.e. the distinct non-zero characteristic roots of $A\Sigma$
h vector of the respective orders of multiplicity $m_i$ of the $\lambda$s
delta the non-centrality parameters $\delta_i$
maxit the maximum number of term $K$ in equation below
eps the desired level of accuracy
mode if mode > 0 then $\beta = mode \times \lambda_{min}$ otherwise $\beta = \beta_B = 2/(1/\lambda_{min} + 1/\lambda_{max})$

Details
Computes $P[Q > q]$ where $Q = \sum_{j=1}^{n} \lambda_j \chi^2(m_j, \delta_j^2)$. $P[Q < q]$ is approximated by $\sum_{k=0}^{K-1} a_k P[\chi^2(m+2k) < q/\beta]$ where $m = \sum_{j=1}^{n} m_j$ and $\beta$ is an arbitrary constant (as given by argument mode).

Value
$Qq$ $P[Q > q]$

Author(s)
Pierre Lafaye de Micheaux (<lafaye@dms.umontreal.ca>) and Pierre Duchesne (<duchesne@dms.umontreal.ca>)

References
P. Duchesne, P. Lafaye de Micheaux, Computing the distribution of quadratic forms: Further comparisons between the Liu-Tang-Zhang approximation and exact methods, Computational Statistics and Data Analysis, Volume 54, (2010), 858-862

Examples
# Some results from Table 3, p.327, Davies (1980)
farebrother(1, c(6,3,1), c(1,1,1), c(0,0,0))
Arguments

q  value point at which the survival function is to be evaluated
lambda  distinct non-zero characteristic roots of \( A\Sigma \)
h  respective orders of multiplicity of the \( \lambda \)s
delta  non-centrality parameters
epsabs  absolute accuracy requested
epsrel  relative accuracy requested
limit  limit determines the maximum number of subintervals in the partition of the given integration interval

Details

Let \( X = (X_1, \ldots, X_n)' \) be a column random vector which follows a multidimensional normal law with mean vector \( 0 \) and non-singular covariance matrix \( \Sigma \). Let \( \mu = (\mu_1, \ldots, \mu_n)' \) be a constant vector, and consider the quadratic form

\[
Q = (x + \mu)' A(x + \mu) = \sum_{r=1}^{m} \lambda_r \chi^2_{h_r, \delta_r}.
\]

The function \( \text{imhof} \) computes \( P[Q > q] \).

The \( \lambda_r \)'s are the distinct non-zero characteristic roots of \( A\Sigma \), the \( h_r \)'s their respective orders of multiplicity, the \( \delta_r \)'s are certain linear combinations of \( \mu_1, \ldots, \mu_n \) and the \( \chi^2_{h_r, \delta_r} \) are independent \( \chi^2 \)-variables with \( h_r \) degrees of freedom and non-centrality parameter \( \delta_r \). The variable \( \chi^2_{h, \delta} \) is defined here by the relation

\[
\chi^2_{h, \delta} = (X_1 + \delta)^2 + \sum_{i=2}^{h} X_i^2,
\]

where \( X_1, \ldots, X_h \) are independent unit normal deviates.

Value

\[
\begin{align*}
Q \ q & \quad P[Q > q] \\
\text{abserr} & \quad \text{estimate of the modulus of the absolute error, which should equal or exceed abs(i-result)}
\end{align*}
\]

Author(s)

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References

P. Duchesne, P. Lafaye de Micheaux, Computing the distribution of quadratic forms: Further comparisons between the Liu-Tang-Zhang approximation and exact methods, Computational Statistics and Data Analysis, Volume 54, (2010), 858-862

Examples

# Some results from Table 1, p.424, Imhof (1961)

# Q1 with x = 2
round(imhof(2,c(0.6,0.3,0.1))$Qq,4)

# Q2 with x = 6
round(imhof(6,c(0.6,0.3,0.1),c(2,2,2))$Qq,4)

# Q6 with x = 15
round(imhof(15,c(0.7,0.3),c(1,1),c(6,2))$Qq,4)

\[ \text{liu} \]

\[ \text{Liu's method} \]

Description

Distribution function (survival function in fact) of quadratic forms in normal variables using Liu et al.'s method.

Usage

\[ \text{liu}(q, \lambda, h = \text{rep}(1, \text{length}(\lambda)), \delta = \text{rep}(0, \text{length}(\lambda))) \]

Arguments

- **q**: value point at which the survival function is to be evaluated
- **lambda**: distinct non-zero characteristic roots of $A\Sigma$, i.e. the $\lambda_i$'s
- **h**: respective orders of multiplicity $h_i$'s of the $\lambda$'s
- **delta**: non-centrality parameters $\delta_i$'s

Details

New chi-square approximation to the distribution of non-negative definite quadratic forms in non-central normal variables. Computes $P[Q > q]$ where $Q = \sum_{j=1}^{n} \lambda_j \chi^2(h_j, \delta_j)$.

This method does not work as good as the Imhof’s method. Thus Imhof’s method should be recommended.

Value

$Qq \quad P[Q > q]$

Author(s)

Pierre Lafaye de Micheaux (<lafaye@dms.umontreal.ca>) and Pierre Duchesne (<duchesne@dms.umontreal.ca>)
References


Examples

# Some results from Liu et al. (2009)
# Q1 from Liu et al.
round(liu(2,c(0.5,0.4,0.1),c(1,2,1),c(1,0.6,0.8)),6)
round(liu(6,c(0.5,0.4,0.1),c(1,2,1),c(1,0.6,0.8)),6)
round(liu(8,c(0.5,0.4,0.1),c(1,2,1),c(1,0.6,0.8)),6)

# Q2 from Liu et al.
round(liu(1,c(0.7,0.3),c(1,1),c(6,2)),6)
round(liu(6,c(0.7,0.3),c(1,1),c(6,2)),6)
round(liu(15,c(0.7,0.3),c(1,1),c(6,2)),6)

# Q3 from Liu et al.
round(liu(2,c(0.995,0.005),c(1,2),c(1,1)),6)
round(liu(8,c(0.995,0.005),c(1,2),c(1,1)),6)
round(liu(12,c(0.995,0.005),c(1,2),c(1,1)),6)

# Q4 from Liu et al.
round(liu(3.5,c(0.35,0.15,0.35,0.15),c(1,1,6,2),c(6,2,6,2)),6)
round(liu(8,c(0.35,0.15,0.35,0.15),c(1,1,6,2),c(6,2,6,2)),6)
round(liu(13,c(0.35,0.15,0.35,0.15),c(1,1,6,2),c(6,2,6,2)),6)
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